Two coupled pendulum →

Let us consider a system of two identical simple pendulums A and B. each pendulum have mass 'm', length of the pendulum is 'l'
A and B are coupled by a linear spring of force constant 'K'

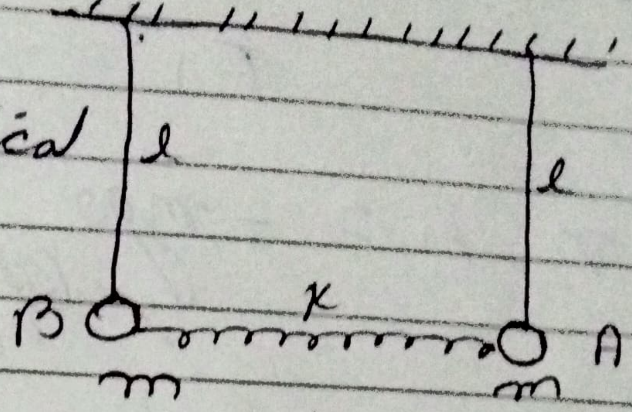

fig (1)

fig (1). Shows the equilibrium position

fig (2) shows the system is slightly disturbed from its equilibrium position

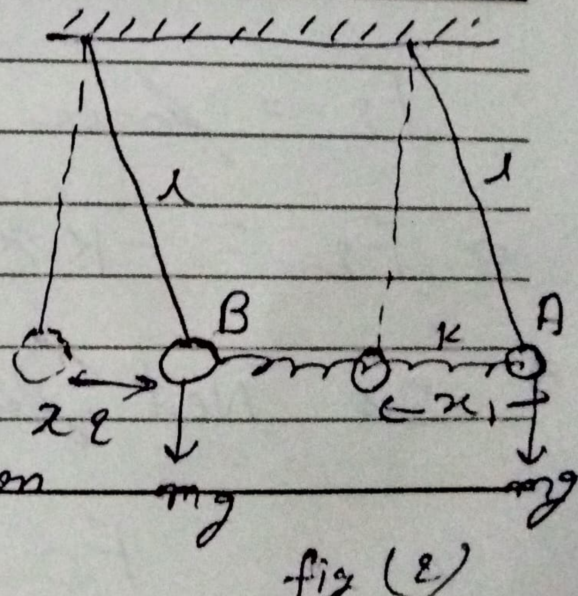

fig (2)

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When the system is disturbed from its equilibrium position and released, the system

Let x_1 and x_2 be the displacement of A and B respectively at an instant of time

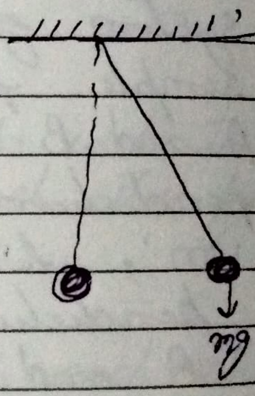
Forces acting on pendulum 'A' are

① When the pendulum is not coupled

$$F_1 = -mg\sin\theta$$

or $F_1 = -mg\theta$

(If θ is small $\sin\theta \simeq \theta$)



$$F_1 = -mg\frac{x}{l} \quad (\because \theta = \frac{x}{l})$$

Force due to attached spring

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F_2 = force constant × displacement

or $F_2 = -Kx$ or $F_2 = -K(x_1 - x_2)$

So Net force acting on 'A'

$$F = -mg\frac{x_1}{l} - K(x_1 - x_2) \quad \text{—(1)}$$

and for pendulum B,

$$F = -mg\frac{x_2}{l} - K(x_2 - x_1) \quad \text{—②}$$

∴ The equation of motion for the pendulum A and B respectively are

$$m\frac{d^2x_1}{dt^2} = -mg\frac{x_1}{l} - K(x_1 - x_2)$$

on dividing both side by 'm' we get

$$\frac{d^2x_1}{dt^2} = -g\frac{x_1}{l} - \frac{K}{m}(x_1 - x_2) \quad \text{—③}$$

and similarly

$$m\frac{d^2x_2}{dt^2} = -g\frac{x_2}{l} - \frac{K}{m}(x_2 - x_1) \quad \text{—④}$$

Put $\frac{g}{l} = \omega^2$ then

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eqn. ③ becomes

$$\frac{d^2x_1}{dt^2} = -\omega^2 x_1 - \frac{K}{m}(x_1 - x_2)$$

eqn ④ becomes

$$\frac{d^2x_2}{dt^2} = -\omega^2 x_2 - \frac{K}{m}(x_2 - x_1)$$

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eq^n 3 can also be written as

$$\frac{d^2x_1}{dt^2} + \omega^2 x_1 = -\frac{K}{m}(x_1 - x_2) \quad \text{——— (5)}$$

eq^n 4 can be also written as

$$\frac{d^2x_2}{dt^2} + \omega^2 x_2 = -\frac{K}{m}(x_2 - x_1) \quad \text{——— (6)}$$

comparing these equations with the equation of SHM – $\frac{d^2x}{dt^2} + \omega^2 x = 0$

It can be That the L.H.S term of the above equation describe the S.H.M

L.H.S term in eq^n 5 and 6 is the normal simple harmonic motion term and R.H.S term is due to the coupling of the spring

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Now adding eq^n 5 and 6 we have

$$\left(\frac{d^2}{dt^2} + \omega^2\right)(x_1 + x_2) = -\frac{K}{m}(x_1 - x_2) + \left(-\frac{K}{m}\right)(x_2 - x_1)$$

$$= -\frac{K}{m}x_1 + \frac{K}{m}x_2 - \frac{K}{m}x_2 + \frac{K}{m}x_1$$

$$\Rightarrow \left(\frac{d^2}{dt^2} + \omega^2\right)(x_1 + x_2) = 0$$

$$\left(\frac{d^2}{dt^2} + \omega^2\right) X = 0 \qquad (\text{where } x_1 + x_2 = X)$$

$$\frac{d^2X}{dt^2} + \omega^2 X = 0 \quad \text{—— (7)}$$

Now subtracting eq^n (5) and (6) we have

$$\left(\frac{d^2}{dt^2} + \omega^2\right)(x_1 - x_2) = \left\{-\frac{k}{m}(x_1 - x_2)\right\} - \left\{-\frac{k}{m}(x_2 - x_1)\right\}$$

$$= -\frac{k}{m}x_1 + \frac{k}{m}x_2 + \frac{k}{m}x_2 - \frac{k}{m}x_1$$

$$= \frac{k}{m}\left[-x_1 + x_2 + x_2 - x_1\right]$$

$$= \frac{k}{m}\left[-2x_1 + 2x_2\right]$$

$$= 2\frac{k}{m}\left[x_2 - x_1\right]$$

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$$\left(\frac{d^2}{dt^2} + \omega^2\right)(r) = 2\frac{k}{m}(x_2 - x_1)$$

(where $x_1 - x_2 = r$
also $x_2 - x_1 = -r$)

$$\frac{d^2r}{dt^2} + \omega^2 r = -\frac{2kr}{m}$$

$$\frac{d^2r}{dt^2} + \omega^2 r + 2\frac{k}{m} r = 0$$

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$$\frac{d^2 Y}{dt^2} + \left(\omega^2 + \frac{2K}{m}\right) Y = 0 \quad \text{--- (8)}$$

eq (7) and (8) have only one variable.